Solution (2) is a finite primitive solution when m is an integer. When m is not an integer or zero, solution (1) gives an infinite series in powers of x. To have integral powers of x, m must equal $-(2n+1)/2$ when n is any integer or zero; also in the original equation $\pm a$ must equal $(2n+3)/2$.

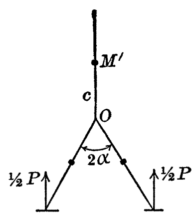
MECHANICS.

293. Proposed by B. F. FINKEL, Drury College.

A man of weight w stands on smooth ice; prove that if, when he gradually parts his legs, kept straight, with his feet in contact with the ice, the pressure of his feet on the ice be constant, his head will descend with uniform acceleration; and that, if f be the acceleration of his head, when his feet exert no pressure on the ice, their pressure on the ice, if f' were the acceleration of his head, would be equal to $\dfrac{f-f'}{f}w$. Walton's *Problems in Theoretical Mechanics*, p. 662.

Solution by E. B. Wilson, Massachusetts Institute of Technology.

We may analyze the man's total mass, M, into M'', the mass of the legs, and M', the remaining mass. The forces acting are W down and P up. Let 2α be the angle between the legs, l their length, $a\cos\alpha$ the distance of their center of gravity below O. Let c be the distance of the center of gravity of M' above O. The head falls through the distance $l(1-\cos\alpha)$. The center of gravity of the whole mass is at a height



$$h=\frac{M'(c+l\cos\alpha)+M''(l-a)\cos\alpha}{M=M'+M''}$$

above the ice. Hence the downward acceleration of the head is

$$f'=-l\frac{d^2\cos\alpha}{dt^2}.$$

The downward acceleration of his C.G. is

$$-\frac{M'l+M''(l-a)}{M}\frac{d^2\cos\alpha}{dt^2}=\frac{W-P}{M}.$$

Hence,

$$\frac{M'l+M''(l-a)}{M}\frac{f'}{l}=\frac{W-P}{M}.$$

If P is constant, then f', which is the only possible variable in this equation, must also be constant. This proves the first part.

If f be the value of f' when $P=0$, we have

$$\frac{M'l+M''(l-a)}{M}\frac{f}{l}=\frac{W}{M}.$$

Hence,

$$\frac{P}{M}=\frac{M'l+M''(l-a)}{M}\frac{(f-f')}{l}$$

and

$$\frac{P}{W}=\frac{f-f'}{f}.$$

This proves the second part.

294. Proposed by EMMA GIBSON, Student at Drury College.

A sphere, revolving about a diameter and not acted on by any extraneous force, expands symmetrically; prove that its vis viva varies inversely as its moment of inertia about its diameter.

Solution by E. B. Wilson, Massachusetts Institute of Technology.

The moment of momentum of the sphere is $I\omega$, where I is the moment of inertia and ω the angular velocity about the axis. This is constant as no external forces are acting. The kinetic energy is $\frac{1}{2}I\omega^2$ or $I^2\omega^2/2I$, which proves the proposition.

MECHANICS.

295. Proposed by B. F. FINKEL, Drury College.

A homogeneous hollow cylinder, whose inner radius is half of its outer radius, rolls without slipping down a plane inclined at an angle α to the horizon. Find its acceleration.

I. Solution by A. M. Harding, University of Arkansas.

The external forces acting are W pounds at the center vertically downward, the reaction normal to the plane, and the friction up the plane.

Let R denote the resultant of the last two and let β denote the angle that its direction makes with the normal. Then the equation of motion of the mass center is

$$W\frac{d^2s}{dt^2}=Wg\sin\alpha-R\sin\beta. \tag{1}$$

If the length of the outer radius of the cylinder is a, then

$$I\frac{d^2\theta}{dt^2}=R\,x\,a\sin\beta=Ra\sin\beta,$$

where the moment of inertia of the cylinder about its axis is $I=\frac{5}{8}Wa^2$.

But, since the cylinder does not slide,

$$s=a\theta.\quad\therefore\ \frac{d^2s}{dt^2}=a\frac{d^2\theta}{dt^2}.$$